# Effect of dephasing on the current statistics of mesoscopic devices

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We investigate the effects of dephasing on the current statistics of mesoscopic conductors with a recently developed statistical model, focusing in particular on mesoscopic cavities and Aharonov-Bohm rings. For such devices, we analyze the influence of an arbitrary degree of decoherence on the cumulants of the current. We recover known results for the limiting cases of fully coherent and totally incoherent transport and are able to obtain detailed information on the intermediate regime of partial coherence for a varying number of open channels. We show that dephasing affects the average current, shot noise, and higher order cumulants in a quantitatively and qualitatively similar way, and that consequently shot noise or higher order cumulants of the current do not provide information on decoherence additional or complementary to what can be already obtained from the average current.

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Current fluctuations in mesoscopic devices are due to the randomness of electron transfer [1] and provide detailed information of the underlying transport mechanisms. From shot noise measurements the distribution functions of open and closed transmission channels in mesoscopic samples like disordered wires [2] and the non-integer charge of quasi-particles [3] are extracted. Such information is not available from conductance measurements and therefore a more complete description of the transport mechanisms can be obtained only by computing the full-counting statistics of the current [4], which comprises the cumulants of all orders. Recently, the possibility to explore additional transport features on cumulants of order higher than the second has been experimentally achieved [5].

A highly debated point in these years is whether additional precious information on the degree of decoherence of a system can be extracted from measurements of shot noise or of higher order cumulants of the current. Several works have addressed this controversial issue in different structures such as electronic Mach-Zehnder interferometers [6, 7, 8], chaotic cavities [9] and resonant tunneling diodes [10, 11, 12, 13].

Indeed, transport in mesoscopic devices is often based on the coherent evolution of the wave function and is therefore very sensible to decoherence caused by the interaction between carriers and the environment [14] that reduces the degree of predictability of system evolution. Hence, the operation of many mesoscopic devices, especially those based on quantum interference, can be undermined even by a small degree of decoherence. The degree of coherence of transport is typically described by a unique synthetic parameter, the dephasing length $L_\phi$, i.e., the characteristic distance over which phase memory is lost. This fundamental physical quantity can be experimentally obtained from conductance measurements on a few suitable devices, such as for example Aharonov-Bohm (AB) rings [15].

The aim of this letter is to explore the influence of dephasing on shot noise and on higher order cumulants of the current in mesoscopic systems like so called "chaotic" cavities and AB rings, in order to verify which cumulants can be conveniently used to gain insight into the degree of coherence of transport.

We adopt a recently developed model [16] that exploits the statistical nature of the dephasing process and has been recently used to investigate magnetoconductance of such structures as a function of the degree of coherence. The method allows us to introduce an arbitrary degree of dephasing in the system and to compute all cumulants of the current, and therefore to investigate their dependence on the dephasing length. We can anticipate that even for a small number of propagating modes, when the correspondence principle cannot be invoked, *essentially all information on dephasing can be obtained from conductance properties*.

Let us highlight the fact that, from the modeling point of view, researchers typically have to simplify the transport model of a mesoscopic conductor, reducing it to the limit of complete coherence, or to the opposite limit of incoherent transport using a semiclassical model, which intrinsically rules out interference effects. Rather intuitively, when the Fermi wave length $\lambda_F$ approaches zero or the number of conducting channels $N_c$ is large, approaches based on completely coherent transport should provide the same result as semiclassical approaches, due to the correspondence principle. In such cases, we can expect that the phase coherence of carriers is irrelevant. As far as shot noise is concerned, for $N_c \gg 1$, such behavior is well known, for example, for the so-called 1/3 suppression of shot noise in diffusive conductors, which has been obtained both with a quantum mechanical description, such as that based on Random Matrix Theory (RMT) [17], or with statistical simulations [18], and in semiclassical terms, using the Boltzmann-Langevin equation [19]. A similar agreement has been obtained for the

so-called 1/4 suppression of shot noise in mesoscopic cavities [20, 21]. However, shot noise and higher order cumulants of the current in the intermediate regime between a fully coherent and incoherent transport have not been determined, leaving unsolved the question of whether they are dependent on the dephasing length.

Ballistic transport in mesoscopic structures is described within the framework of the Landauer-Büttiker theory [22], which does not allow to include directly the effects of dephasing. Such effects are usually treated with phenomenological models, which are based on the insertion of a virtual voltage probe [23] into the ballistic region: electrons traveling from source to drain can be absorbed by a third probe, where they lose their phase information, and then are re-injected into the conductor.

An alternative phenomenological model which describes decoherence as a phase randomizing statistical process has been recently proposed and implemented with a Monte Carlo method by the authors [16]. Such method treats decoherence as a random fluctuation of the phase of the propagating modes involved in the computation of the scattering matrix (S-matrix), and enables us the obtain cumulants of the current from Monte Carlo simulations over a sufficiently large ensemble of runs [24].

Current fluctuations in the leads are related to the number of particles $n$ that traverse the devices during the observation time $t$. For example, the first cumulant $\langle\langle n \rangle\rangle = \langle I \rangle t/e$ gives the mean current $\langle I \rangle$ ($e$ is the absolute value of the electron charge). The second cumulant $\langle\langle n^2 \rangle\rangle = St/2e^2$ gives the shot noise power spectral density $S$. Consequently, the Fano factor $F$, defined as the ratio of $S$ to the power spectral density of a Poissonian process $2eI$ can be written as the ratio of the second to the first cumulant: $F = \langle\langle n^2 \rangle\rangle / \langle\langle n \rangle\rangle$. From a numerical point of view, the transmission matrix is computed by dividing the domain in $N_x$ slices, then by calculating the scattering matrix $s_i$ for each slice, and by finally composing all the partial S-matrices with the appropriate rules [25] in order to obtain the complete S-matrix of the conductor $s_T = s_1 \otimes s_2 \otimes \cdots \otimes s_{N_x - 1}$. We include dephasing as a statistical process, by adding a random term to the phase accumulated by each propagating mode in each slice. Average quantities are obtained performing simulations on a sufficiently large ensemble of runs (of the order of hundreds). The random term added to each mode of the $i$-th slice obeys a Gaussian probability distribution of zero average and variance $\sigma_i^2$ determined by the length of the slice $\Delta x_i$ and by the dephasing length $L_\phi$ as $\sigma_i^2 = \Delta x_i / L_\phi$. By varying $L_\phi$ we are able to explore the complete range of transport regimes, from completely coherent to completely incoherent.

We consider systems of non interacting electrons at zero temperature, and all aspects can be expressed in terms of the eigenvalues $T_n$ of the transmission matrix $T = t^\dagger t$. Indeed, the $k$-th order cumulants $\langle\langle n^k \rangle\rangle$ ($k$ is a non-zero integer) of the number of transmitted particles are defined as the coefficients of the series expansion of the logarithm of the characteristic function $\chi(\lambda)$ [1]:

$$\ln \chi(\lambda) = \sum_k \frac{(i\lambda)^k}{k!} \langle\langle n^k \rangle\rangle \, . \tag{1}$$

Due to the microscopic mechanism of transport, the probability to have $m$ transmitted electrons through a generic channel is given by the binomial distribution $P_m = C_N^m T^m (1-T)^{N-m}$, where $N$ is the average number of electrons that attempt to traverse the device[26, 27]. The characteristic function is given by $\chi(\lambda) = \sum_m P_m \exp(im\lambda) = [T \exp(i\lambda) + 1 - T]^N$, which in the case of several independent channels reads

$$\chi(\lambda) = \prod_j [T_j \exp{(i\lambda)} + 1 - T_j]^N \, , \tag{2}$$

where the product is performed over all transmission channels in the conductor. From Eq. (2) we can obtain the following expression for the cumulant of order $k$ [27]:

$$\langle\langle n^k \rangle\rangle = N \sum_j \left[ T(1-T) \frac{d}{dT} \right]^{k-1} T \Bigg|_{T=T_j} \, . \tag{3}$$

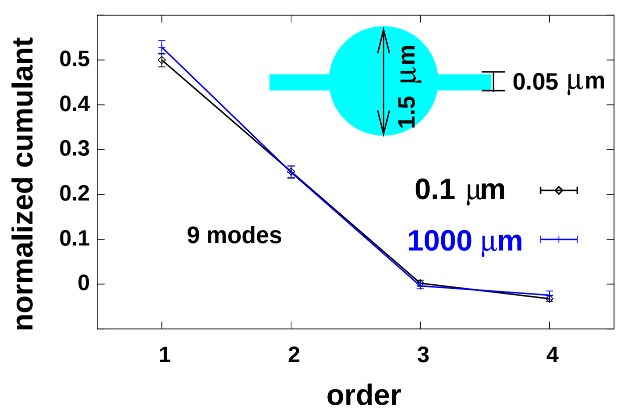


FIG. 1: (Color online) Cumulants $\langle\langle n^k \rangle\rangle$ for the cavity shown in the inset for $k = 1, 2, 3, 4$, when the Fermi energy allows $N_c = 9$ conducting modes in the leads. The two curves correspond to the cases of $L_\phi = 0.1$ $\mu$m (incoherent transport), and $L_\phi = 1000$ $\mu$m (coherent transport). For $k = 2, 3, 4$ the cumulants are normalized to $\langle\langle n \rangle\rangle$. The first order cumulant is normalized to $N_c$. The diameter of the circular cavity is 1.5 $\mu$m and the lead width is 50 nm.

As a first case, we focus on a so called "chaotic" cavity, whose structure is shown in the inset of Fig. 1, and consisting of a circular cavity with diameter of 1.5 $\mu$m and lead width of 50 nm. We consider the material properties of GaAs. The dwell time of electrons is large enough to provide the one fourth suppression of shot noise expected for such a structure [28]. This condition is sufficient to consider negligible the trajectories responsible of back reflection into the lead.

We explore the dependence of the transport properties of the cavity on the strength of the decoherence mechanism. We consider first the two limiting cases of $L_\phi = 10^{-1}$ $\mu$m, which is much smaller than the sample size and therefore corresponds to an almost fully incoherent regime, and of $L_\phi = 10^3$ $\mu$m, which is much larger than the length of the classical path covered by the electrons inside the cavity, and therefore corresponds to coherent transport. In Fig. 1 we show that first four cumulants for $N_c = 9$, obtained from Eq. (3), are almost identical in the two cases. The total conductance is half the conductance of a single constriction, and the Fano factor is 1/4 in both cases.

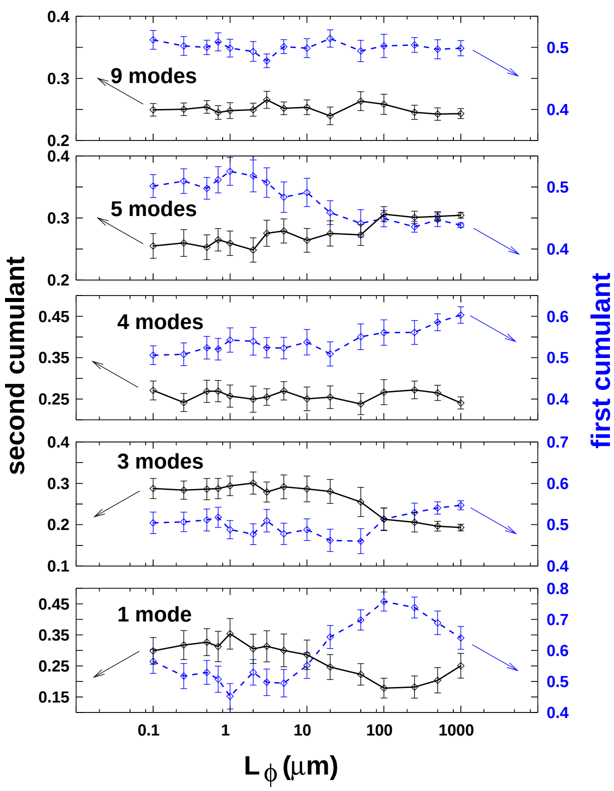


FIG. 2: (Color online) Normalized second cumulant $\langle\langle n^2\rangle\rangle/\langle\langle n\rangle\rangle$ or Fano factor (solid line) of the quantum dot shown in Fig. 1 compared with the normalized first cumulant $\langle\langle n\rangle\rangle/N_c$ (dashed line) as a function of the dephasing length $L_\phi$. From the bottom to the top the Fermi energy allows the conduction of 1, 3, 4, 5, and 9 modes. The error bars in the plot are $\pm 2s_{\rm m}$, where $s_{\rm m}$ is the standard deviation of the sample average.

The fact that quantum coherence does not influence the transport properties for a large number of conducting channels in the structure is simply consistent with the correspondence principle. Hence, we investigated the dependence of the second and of the fourth cumulant on the dephasing strength, for a smaller number of conducting channels in the leads.

In Fig. 2 and Fig. 3 results are shown for $N_c = 1, 3, 4, 5, 9$ propagating modes in the leads, and a much larger number of corresponding open channels in the cavity. In such figures we have plotted the first cumulant of the current $\langle\langle n\rangle\rangle/N_c$, together with the normalized second cumulant $\langle\langle n^2\rangle\rangle/\langle\langle n\rangle\rangle$ and the normalized fourth cumulant $\langle\langle n^4\rangle\rangle/\langle\langle n\rangle\rangle$, as a function of $L_\phi$.

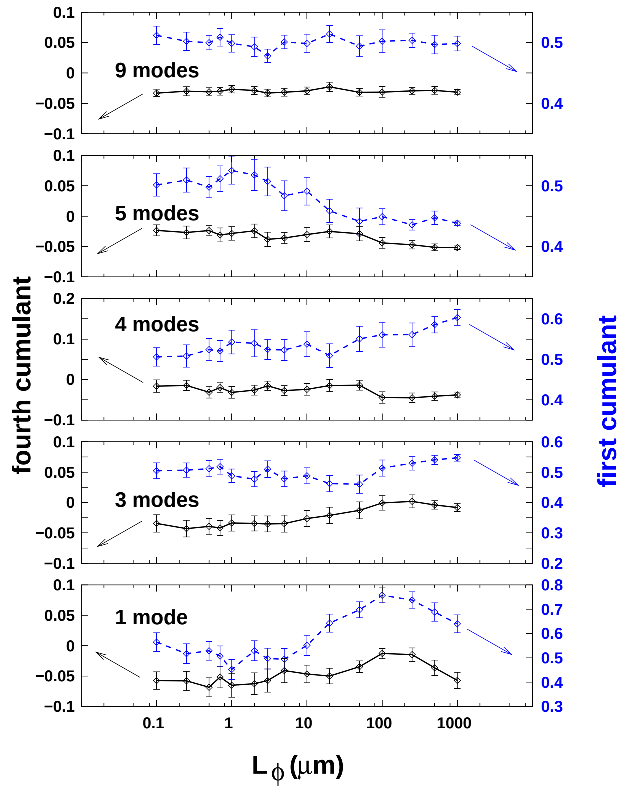


FIG. 3: (Color online) Normalized fourth cumulant $\langle\langle n^4\rangle\rangle/\langle\langle n\rangle\rangle$ (solid line) compared with normalized first cumulant $\langle\langle n\rangle\rangle/N_c$ (dashed line) of the cavity shown in Fig. 1 as a function of the dephasing length $L_\phi$. From the bottom to the top the Fermi energy allows the conduction of 1, 3, 4, 5, 9 modes, respectively.

The figures show very clearly that both the second and fourth order cumulants are largely independent of the dephasing length, within the range of the error bars due to the finite ensembles considered. In the case of large $N_c$ the theoretical values of both $\langle\langle n^2\rangle\rangle/\langle\langle n\rangle\rangle = 1/4$ and $\langle\langle n^4\rangle\rangle/\langle\langle n\rangle\rangle = -1/32$ are recovered.

In addition, for small $N_c$, whenever a modulation of the values of the second or fourth order cumulants can be observed as a function of $L_\phi$, a corresponding modulation with comparable or larger amplitude can be observed in the first cumulant. In our view this is a demonstration that higher order cumulants do not provide additional information on dephasing with respect to that already provided by conductance. Even in the case of single mode transmission in the lead, different cumulants are altered by the degree of coherence essentially in the same way.

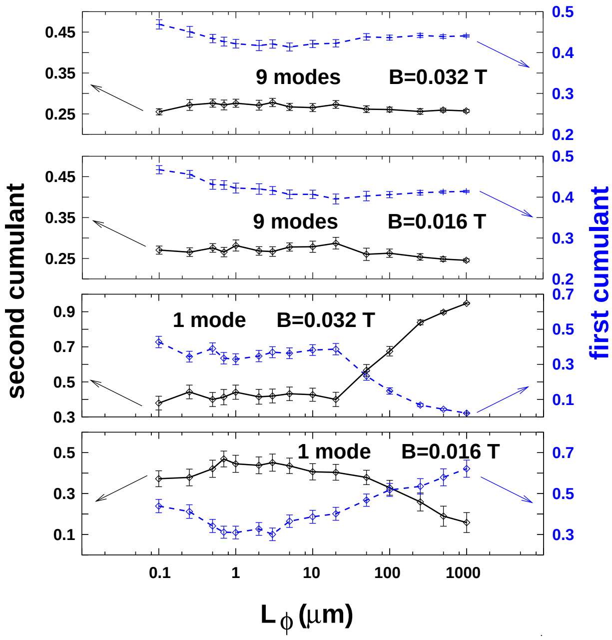


FIG. 4: (Color online) Normalized second cumulant or Fano factor (black solid line) of the AB ring as a function of the dephasing length compared with the normalized first cumulant $\langle\langle n \rangle\rangle/N$ (blue dashed line). From the top to the bottom we sketch the cases corresponding to ($N_c = 9, B = 0.032$ T), ($N_c = 9, B = 0.016$ T), ($N_c = 1, B = 0.032$ T), and ($N_c = 1, B = 0.016$ T). The two values of the magnetic field are such that the magnetoconductance presents a maximum (for $B = 0.016$ T) and a minimum ($B = 0.032$ T) in the AB oscillations. The device is obtained from the cavity sketched in Fig. 1 by inserting a central antidot with diameter of 0.9 $\mu$m (not shown).

The second device we consider is an Aharonov-Bohm ring, that exhibits very regular magnetoconductance oscillations due to quantum interference, and is therefore very sensitive to the effects of decoherence. In Fig. 4 we show simulation results for the Fano factor of an Aharonov-Bohm ring, for the cases corresponding to two values of a perpendicular magnetic field corresponding to a maximum ($B = 0.016$ T) and a minimum ($B = 0.032$ T) of magnetoconductance. As can be seen, for the two limiting cases of a small ($N_c = 1$) and a large ($N_c = 9$) number of conducting channels, the Fano factor depends on the dephasing length $L_\phi$ in the same way as the conductance, and therefore does not provide additional insights on decoherence.

In this work we have investigated the influence of dephasing on the transport properties of mesoscopic structures in order to evaluate the possibility to achieve information on the degree of decoherence from shot noise properties or from higher order cumulants of the current. We have used a recently developed statistical model to include a distributed arbitrary level of dephasing in the device. We have focused on two types of structures with a varying number of propagating channels that we believe to be representative of the broad class of mesoscopic devices. Our conclusion is that no additional information on the degree of dephasing is to be expected with respect to that already provided by the conductance.

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